Total No. of printed pages = 8

MATHEMATICS – I

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answers : $1 \times 5 = 5$

    (a) Square Root of $15 - 8i$ is

    (i) $4 + i$ (ii) $4 - i$

    (iii) $\pm (4 - i)$ (iv) $4 \pm i$

    (b) Sum of cube roots of unity is

    (i) 1   (ii) 0

    (iii) -1 (iv) 3

(c) Middle term in $\left(x-\dfrac{1}{2x}\right)^6$ is

    (i)   20   (ii) – 20

    (iii) $-\dfrac{20}{8}$ (iv) $\dfrac{20}{8}$

(d) Value of $\left(2+\sqrt{3}\right)^3+\left(2-\sqrt{3}\right)^3$ is

    (i)   50   (ii) 52

    (iii) $5\sqrt{3}$ (iv) $3\sqrt{3}$

(e) For $\log_a 16 = -4$, value of a is

    (i)   2   (ii) 4

    (iii) $\dfrac{1}{3}$ (iv) $\dfrac{1}{2}$

2. Fill in the blanks :   1×4=4

    (a) The value of $^5C_4 \times {}^3P_1$ is _____.

    (b) If 4 parallel lines in a plane are intersected by 6 parallel lines, the number of parallelogram formed is _____.

(c) $\begin{pmatrix} 1 & 2 \\ -2 & 0 \\ 3 & 1 \end{pmatrix} + 2\begin{pmatrix} 3 & -2 \\ 2 & 1 \\ -1 & 0 \end{pmatrix} = $ ————.

(d) The third order unit matrix is _____.

3. Write True or False :   1×6=6

    (a) If $\cos\theta=\dfrac{4}{5}$, then $\tan\theta=\dfrac{1}{5}$

    (b) $\sin^2 24° + \sin^2 66° = 2$

    (c) $\tan (A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

    (d) $\sin 2A=\dfrac{2\tan A}{1+\tan^2 A}$

    (e) $\tan^{-1}x + \cot^{-1}\dfrac{1}{x} = 2\tan^{-1}x$

    (f) $\sin^{-1} \tan\left(\dfrac{3\pi}{4}\right)=\dfrac{3\pi}{4}$.

4. (a) Draw Argand diagram of   2

       (i) $-6i$ (ii) $3 + i$

   (b) Express as sum of simple algebraic fractions (any *two*) :   2+2=4

       (i) $\dfrac{2x-3}{x^2+6x+8}$ (ii) $\dfrac{2}{x^2-9}$

       (iii) $\dfrac{1}{x\left(x^3+1\right)}$

   (c) (i) In how many ways the letters of the word DAUGHTER be arranged without changing the order of the vowels in the word ?   2

            Or

      (ii) Out of 9 players and 13 singers how many different committees can be formed, each consisting of 3 players and 5 singers ?   2

   (d) If $a^2 + b^2 = 7ab$, prove that   2

      $2 \log (a-b) = \log 5 + \log a + \log b$.

5. (a) Expand up. to 4th term : $(1 + 2x)^{-4}$ 2

   (b) (i) If $\dfrac{1}{x+y}, \dfrac{1}{2y}, \dfrac{1}{y+z}$ are in AP, prove that x, y, z are in GP.   3

            Or

      (ii) Write the series for e^x and e^{-x} Prove that :   3

$$\frac{1}{2}\left(e^x + e^{-x}\right) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\ldots$$

   (c) If $A = \begin{pmatrix} 1 & -2 & 3 \\ 4 & -1 & 2 \\ -2 & 3 & 0 \end{pmatrix}$ find det (A).   2

   (d) Evaluate (any *two*) :   2+2=4

      (i) $\begin{vmatrix} 21 & 4 & -6 \\ 9 & 7 & 2 \\ 0 & -3 & 5 \end{vmatrix}$

      (ii) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

(|iii) $\begin{vmatrix} 13 & 3 & 23 \\ 30 & 1 & 7 \\ 39 & 9 & 69 \end{vmatrix}$

6. (a) Answer any *one* : $2 \times 1 = 2$

     (i) If $\sin^4 \theta + \sin^2 \theta = 1$, prove that $\tan^4 \theta - \tan^2 \theta = 1$.

     (ii) Solve $-2 \cos^2\theta + \sin\theta = 1$.

   (b) If $A + B + C = \pi$, prove that

     $\cos A + \cos B + \cos C = 1 + 4 \sin \dfrac{A}{2}$

     $\sin \dfrac{B}{2} \sin \dfrac{C}{2}.$ 3

   (c) Answer any *one* : $2 \times 1 = 2$

     (i) Prove that $\tan^{-1}x + \sin^{-1}x = \tan^{-1}$

$$\frac{x\left(1 + \sqrt{1 - x^2}\right)}{\sqrt{1 - x^2} - x^2}$$

     (ii) If $\tan^{-1}x = 3$, find $\cos^{-1}x$.

   (d) Prove that $2 \tan^{-1}\dfrac{1+x}{1-x} + \sin^{-1}\dfrac{1-x^2}{1+x^2} = \pi.$ 3

(e) (i) Apply cosine formula to prove $\cos \dfrac{c}{2}$

      $= \sqrt{\dfrac{s(s-c)}{ab}}.$

     (ii) Write sine Rule and cosine Rule for a triangle. $2+2=4$

7. (a) A river is 96m wide. Its depth at a distance of x m from one bank is y m and is given as in the following table :   3

x :	0	12	24	36	48	60	72	80	96
y :	1	3	5	10	12	8	6	3	2

   (b) Answer any *two* : $2+2=4$

     (i) A cuboidal water tank is 2m long, 3m wide, 4m deep. Find the volume of the water tank.

     (ii) The height of a cylinder is 10 cm and the ratio of its volume to the lateral surface area is 3:2. Find its radius.

     (iii) Find the volume of the largest sphere that can be cut out of a cube of volume 27 cubic cm.

(c) The frustum of a square pyramid is 15m high, the top base has area 5 sq.m. and bottom base has 9 sq.m. The top is capped by a pyramid of height 1m. Find the volume of the solid. 3

8. (a) (i) The vertices of a triangle are (5, 2), (3, 6), (0, 0), find its area.

(ii) Find equation to the straight line having gradient –2 and passing through (5, 1).

2+2=4

(b) Answer any two : 3+3=6

(i) Find locus of the point moving at equal distance from (3, –1) and (–2, 5).

(ii) A straight line is $3x - 2y + 1 = 0$. Find its distance from the point (1, 3). Write its intercepts on x–axis. Draw the line.

(iii) Find equation to the line parallel to $x - 5y + 3 = 0$ and passing through (4, 0).